Introduction to Dynamic System Stability

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COMP 517

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Definitions

- Global stability
  - Function stays within a certain range of equilibrium
- Asymptotic stability
  - Function decays to equilibrium point
- Exponential stability
  - Function decays exponentially

Linear Stability Criteria

- Pole placement
  - All eigenvalues in LHP

- Routh-Hurwitz
  - SISO only
  - Gives number of positive roots

Nonlinear Criteria

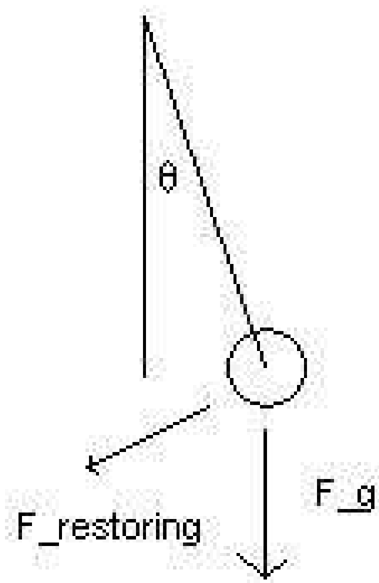
- Lyapunov Candidate Function, V(x)

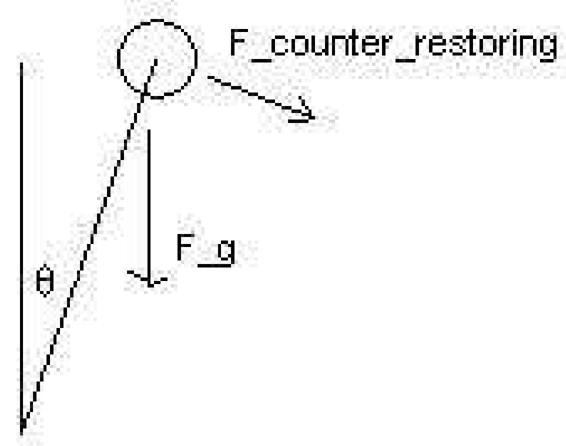
  – Positive definite $V(x) \geq 0 \quad \forall\, x$

  – Derivative is negative definite $\dot{V}(x) \leq 0 \quad \forall\, x$

- Generally approximates the energy content of a system

Example: Some Pendulums

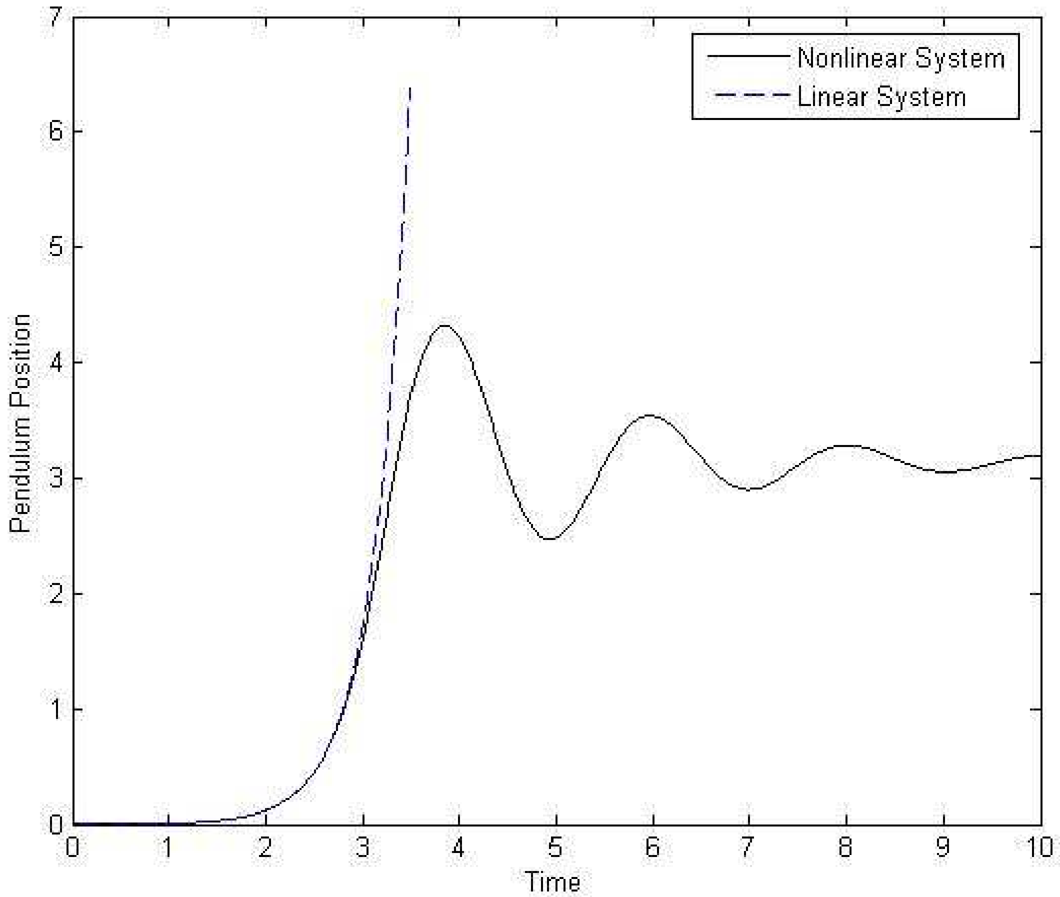




$$\ddot{\theta} + \frac{g}{\ell}\sin\theta = F_i \qquad \ddot{\theta} - \frac{g}{\ell}\sin\theta = F_i$$

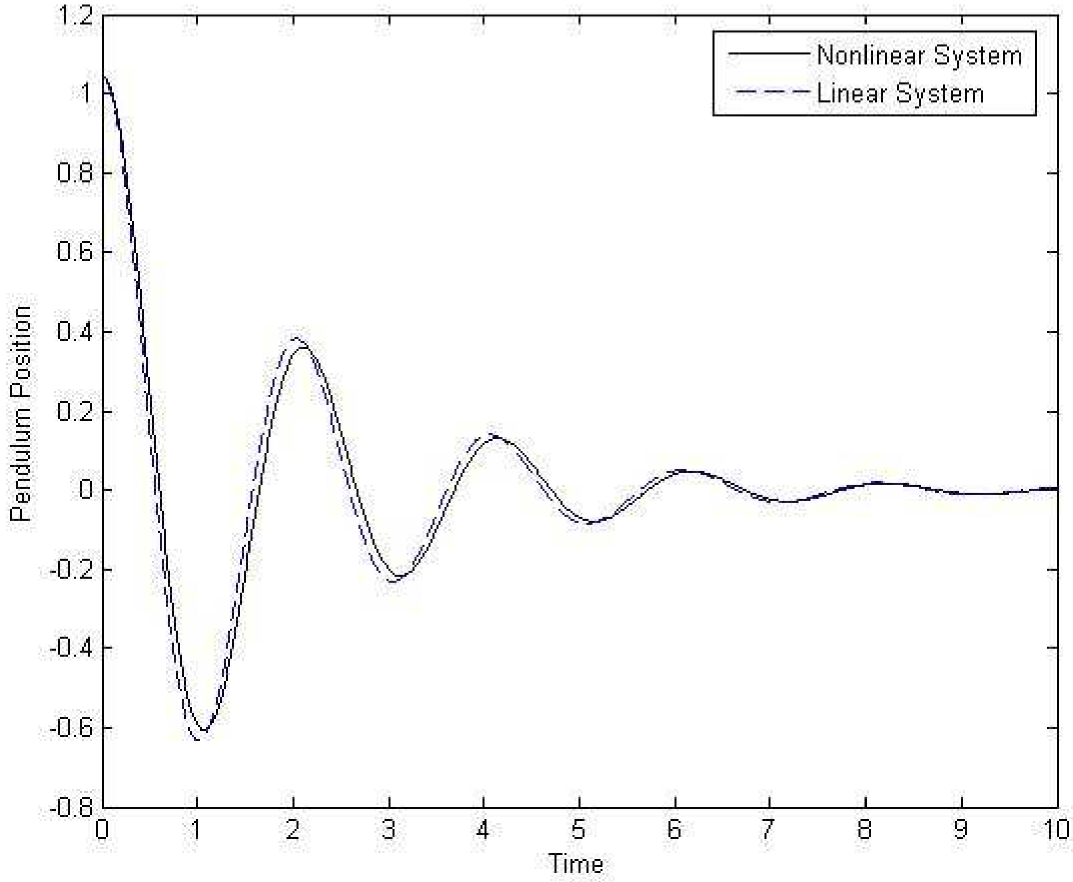
$$F_i = -K_p\theta - K_d\dot{\theta}$$

Unstable Pendulum Behavior



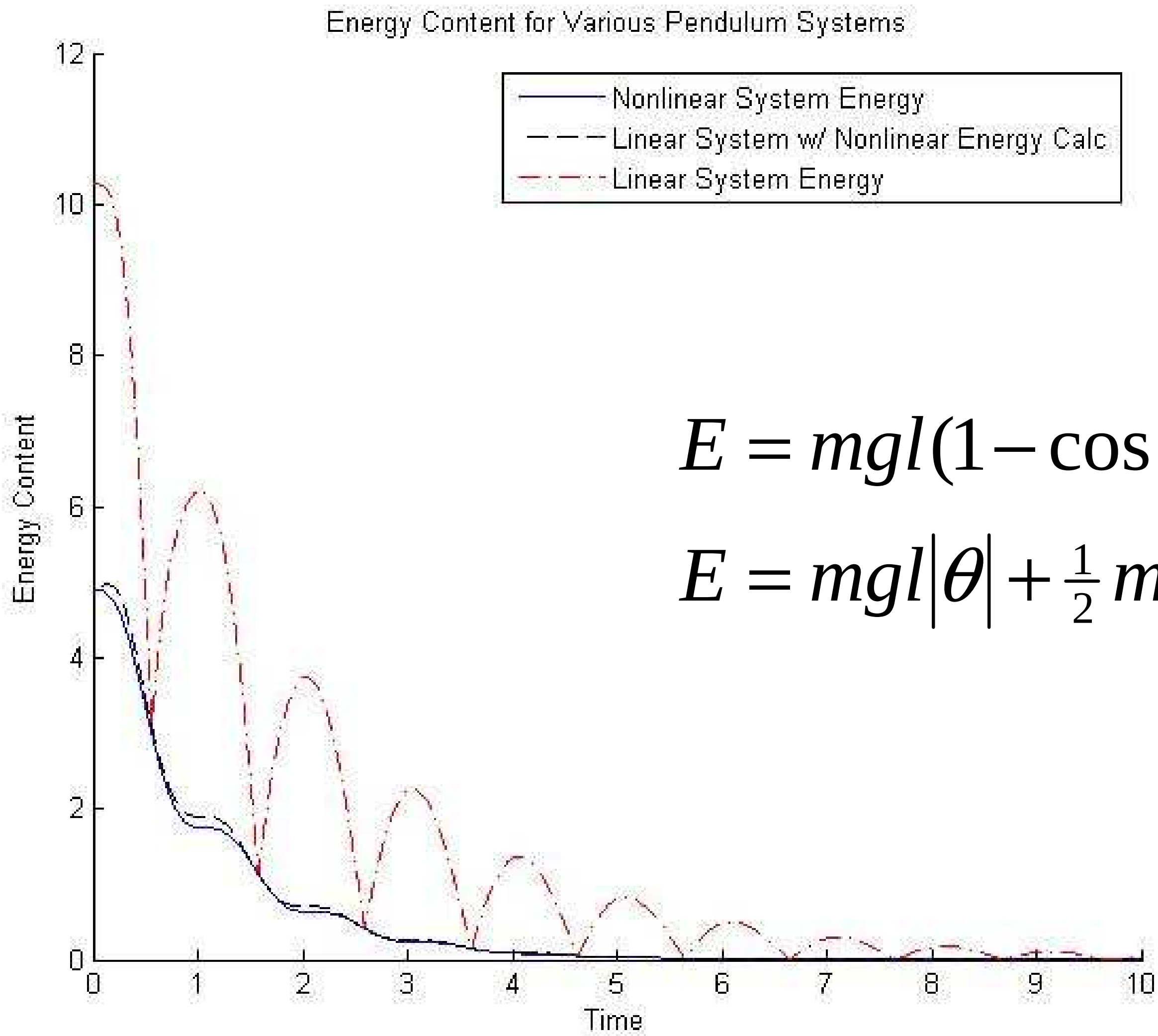
$K_p = 0$, $K_d = 1$, $l = 1$, $g = 9.8$, $\theta(0) = .001$, $\theta'(0) = 0$

Stable Pendulum Behavior



$K_p = 0$, $K_d = 1$, l = 1, g = 9.8, theta(0) = π/3, theta'(0) = 0

System Energy



$$E = mgl(1 - \cos\theta) + \frac{1}{2} m\ell^2 \dot{\theta}^2$$

$$E = mgl\left|\theta\right| + \frac{1}{2} m\ell^2 \dot{\theta}^2$$